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INFLUENCE OF THE METHODS OF CONSTRUCTING EPHEMERIDES OF MAJOR PLANETS AND THE MOON ON THE ACCURACY OF PREDICTING MOTION OF ASTEROIDS

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The jump-like behavior of coordinates of planets and the Moon as well as of their derivatives retrieved from modern ephemerides is demonstrated. Discontinuities of the coordinates and derivatives take place at the junctions of the adjacent interpolation intervals each of which in the ephemerides has its own set of the coefficients of the Chebyshev polynomials. This is demonstrated on an example of the ephemerides DE431 and EPM2011. The precision of predicted motion of asteroids is estimated with allowance for perturbations from the ephemerides DE431 and EPM2011. It is demonstrated that the step of numerical integration of the equations of motion must be adjusted to the junctions of the ephemeris intervals; in this case, the precision of integration increases by several orders of magnitude. In addition, to eliminate discontinuities of the coordinates and of their first derivatives arising in calculations with quadruple precision, an algorithm of smoothing ephemerides at the junctions of interpolation intervals is developed that allows the discontinuities of the coordinates and their derivatives up to any preset order to be eliminated. The algorithm is used to smooth the ephemerides DE431 and EPM2011 up to the fourth-order derivatives. It is demonstrated that in calculations with the quadruple precision, the application of the smoothed ephemerides allows the accuracy of numerical integration to be increased approximately by 10 orders of magnitude.

Keywords: DE431, EPM2011, discontinuities of derivatives, smoothing.

In [1–4] the author has published results of investigations into the problem of increasing precision of numerical prediction of asteroid motion using modern ephemerides published by the Jet Propulsion Laboratory (USA) [5–7] and Institute of Applied Astronomy of the Russian Academy of Sciences [8, 9] to take into account perturbations from major planets and the Moon. In [1–4], the jump-like behavior of the coordinates of planets and their derivatives retrieved from ephemerides DE405, DE406, DE408, DE421, DE422, DE423, and DE430 was demonstrated, and methods of eliminating the influence of this behavior on the precision of predicted motion of asteroids were considered. In the present work, results of analogous investigations are presented for the latest ephemerides DE431 [7] and EPM2011 [8, 9] published by the present time.

Recall that in modern ephemerides of the major planets and the Moon, the information on the coordinates of objects is stored in compressed form based on the interpolation Chebyshev polynomials. Structures of different ephemerides as a whole are identical: all time interval covered by the ephemerides is subdivided into a fixed number of small interpolation intervals with a fixed set of the coefficients of the Chebyshev polynomials for each interval. Moreover, at the junctions of the adjacent intervals, the continuity of the interpolated coordinates and their first left and right derivatives retained, but the derivatives of the second and higher orders are discontinuous. In addition, in all ephemerides published by the present time the coefficients of the Chebyshev polynomials are given only with 16

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decimal places, that is, in the form of 64-bit computer numbers with floating point (the so-called double-precision numbers). Therefore, the interpolated coordinates and their first derivatives are continuous at the junctions of the adjacent intervals only for calculations with double precision. However, calculations with increased precision are more often used nowadays, for example, using 80-bit computer numbers with floating point which have 19 decimal places [10, 11] or 128-bit numbers with 34 decimal places (the so-called quadruple precision numbers). Such high computer precision is necessary, for example, to search for asteroid orbits leading to a collision with the Earth [12–14]. In this case, the interpolated coordinates and their first derivatives are discontinuous at the junctions of the ephemeris intervals approximately in the 15th–16th decimal places.

The above-indicated discontinuities are inevitably manifested through the behavior of the right sides of the differential equations describing asteroid motion, since the right sides depjunction continuously on the coordinates of the perturbing bodies. Therefore, they are also discontinuous at the junctions of the ephemeris intervals, namely, the second- and higher-order derivatives are discontinuous in double-precision calculations; in calculations with higher-order precision, their right sides and their first, second, etc. derivatives are discontinuous. Such behavior of the right sides decreases the precision of numerical integration of the equations of motion in both cases, since all methods of numerical integration, as a rule, suggest continuity of functions in the right sides and their smoothness up to the serial order as high as possible.

Nowadays there are two approaches to the construction of ephemerides for planets and the Moon. According to the first approach used in the American ephemerides DE, the coefficients of the Chebyshev polynomials required for interpolation of only coordinates of objects are given in ephemerid files. To obtain the derivatives of the coordinates (for example, velocities or accelerations), the expressions for the interpolated coordinates must be differentiated the required number of times. The second approach is used in Russian ephemerides EPM [8, 9], according to which coefficients for interpolation of object velocities are given in the ephemerides, coordinates of these objects are calculated by integration of expressions for their velocities, and integration constants are also given in the ephemerides. The derivatives of the velocities are calculated in the same way as in the American ephemerides.

Let us consider the first approach to the ephemerid construction. The expression for any interpolated coordinate (designated by x) and its derivatives has the form

$$x^{(k)}(\tau) = a_0 p_0^{(k)}(\tau) + \ldots + a_n p_n^{(k)}(\tau)\,, \tag{1}$$

where p_i are the Chebyshev polynomials of degree i, a_i are the coefficients given in the ephemerides, n is the number of the coefficients for the given coordinate minus unity, k is the serial order of the derivative ($k = 0$ corresponds to the coordinate itself), and τ is the normalized time varying from –1 to +1 in the interpolation interval. The Chebyshev polynomials and their derivatives in Eq. (1) are calculated from the recurrent relations [3]

$$\begin{aligned}
&p_0 = 1,\ \ p_1 = \tau,\ \ p_i = 2\tau p_{i-1} - p_{i-2} \quad (i = 2, \ldots, n),\\
&p_0' = 0,\ \ p_1' = 1,\ \ p_i' = 2(p_{i-1} + \tau p_{i-1}') - p_{i-2}' \quad (i = 2, \ldots, n),\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&p_0^{(k)} = 0,\ \ p_1^{(k)} = 0,\ \ p_i^{(k)} = 2(kp_{i-1}^{(k-1)} + \tau p_{i-1}^{(k)}) - p_{i-2}^{(k)} \quad (i = 2, \ldots, n;\ \ k \ge 2).
\end{aligned} \tag{2}$$

Within the limits of the second approach, the velocity components are calculated by the formula analogous to formula (1). Therefore, having designated by v any of the three velocity components, we can write

$$v^{(k)}(\tau) = a_0 p_0^{(k)}(\tau) + \ldots + a_n p_n^{(k)}(\tau)\,. \tag{3}$$

The expression for the coordinates is obtained by integration over time of formula (3) at $k = 0$:

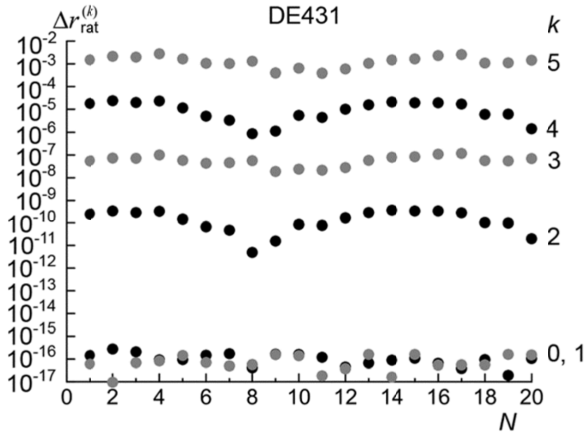

Fig. 1. Relative magnitudes of jumps of the interpolated coordinates.

$$x(\tau) = C + \frac{T}{2}(a_0 w_0(\tau) + \ldots + a_n w_n(\tau)), \tag{4}$$

where C is the integration constant equal to the value of the coordinate x at $\tau = 0$, i.e., in the middle of the interpolation interval of duration T given in the ephemerides together with the coefficients a_i, and w_i are the primitive functions of the Chebyshev polynomials p_i calculated in terms of the polynomials themselves using the relationships (ftp://quasar.ipa.n w.ru/incoming/EPM/iaa-ephemerides-short-description.pdf)

$$w_0 = \tau,\ \ w_1 = (p_0 + p_2)/4,$$
$$w_{2i} = \frac{1}{2}\left(\frac{p_{2i+1}}{2i+1} - \frac{p_{2i-1}}{2i-1}\right),\ \ w_{2i+1} = \frac{1}{2}\left(\frac{p_{2i+2}}{2i+2} - \frac{p_{2i}}{2i}\right) + \frac{(-1)^i}{4}\left(\frac{1}{i} + \frac{1}{i+1}\right)\ \ (i = 1, \ldots, [n/2]). \tag{5}$$

Here the square brackets denote the integer part of the number. For even n values, the last expression in Eqs. (5) for $i = [n/2]$ is not used.

Let us now demonstrate the jump-like behavior of the planet coordinates and their derivatives at the junctions of the adjacent interpolation intervals in the ephemerides DE431 and EPM2011. Values of the coordinates and their left and right derivatives at any arbitrary junction are calculated from formulas (1) and (4) using the corresponding set of the ephemeris coefficients a_i and substitution of τ values: +1 for the left set of the coefficients a_i and –1 for the right set. We designate the calculated left and right coordinates and their derivatives by $x_L^{(k)}, y_L^{(k)}, z_L^{(k)}$ and $x_R^{(k)}, y_R^{(k)}, z_R^{(k)}$, respectively. The relative jump size at the junctions of the interpolation intervals is

$$\Delta r_{\mathrm{rat}}^{(k)} = \sqrt{\frac{(x_R^{(k)} - x_L^{(k)})^2 + (y_R^{(k)} - y_L^{(k)})^2 + (z_R^{(k)} - z_L^{(k)})^2}{(x_m^{(k)})^2 + (y_m^{(k)})^2 + (z_m^{(k)})^2}}, \tag{6}$$

where $x_m^{(k)} = (x_R^{(k)} + x_L^{(k)})/2$, $y_m^{(k)} = (y_R^{(k)} + y_L^{(k)})/2$, and $z_m^{(k)} = (z_R^{(k)} + z_L^{(k)})/2$. Formula (6) is convenient, because it shows in what decimal place the jump is observed, since it is its first nonzero decimal place.

Figure 1 shows results of calculations of $\Delta r_{\mathrm{rat}}^{(k)}$ for the barycenter of the Earth-Moon system at 20 successive junctions of the interpolation intervals chosen randomly near the center of the corresponding ephemerides (the serial number N of the junction is plotted on the horizontal axis). The k values changed from 0 to 5, that is, jumps of the coordinates and their derivatives up to the fifth order were calculated. For the ephemerides EPM2011, k in Fig. 1 also denotes the serial order of the derivative of the coordinates rather than of the velocities, as in formula (3). All calculations were carried out with quadruple precision, the required computing programs were written by the author except for a small part of the program code devoted to calculations of the primitive functions for the Chebyshev polynomials given by Eq. (5) that was borrowed from (ftp://quasar.ipa.n w.ru/incoming/EPM/Fortran/calc_eph. F).

As can be seen from Fig. 1, the coordinates and their first derivatives in the ephemerides DE431 have about the same jumps in the 16^{th}–17^{th} decimal places, the second derivatives jump in the 10^{th}–12^{th} decimal places, the third derivatives jump in the 7^{th}–8^{th} decimal places, the fourth derivatives jump in the 5^{th}–6^{th} decimal places, and the fifth derivatives jump in the 2^{nd}–3^{rd} decimal places. Jumps of the higher-order derivatives become comparable with their values and are not shown in Fig. 1. As to the ephemerides EPM2011, as can be seen from Fig. 1, the behavior of the coordinates and their first derivatives differs: whereas the coordinates jump approximately in the 15^{th}–16^{th} decimal places, and their first derivatives jump already in the 13^{th}–14^{th} decimal places. Jumps of higher-order derivatives in both ephemerides were approximately identical.

Such behavior of the coordinates and their derivatives is characteristic not only of the barycenter of the Earth-Moon system, but also of all other objects in the ephemeris data; therefore, no illustrations for them are presented here. We note that such behavior is peculiar to all American ephemerides and has been demonstrated, for example, in [2, 4] for the ephemerides DE422 and DE430 [5].

Since in double-precision calculations the coordinates of ephemeris objects and their first derivatives are practically continuous and only the derivatives of the second and higher orders are discontinuous, an obvious method of elimination of the influence of these discontinuities on the precision of numerical integration is adjustment of the integration step to the junctions of the ephemeris intervals. Such adjustment can be implemented by additional correction of the integration step [2] consisting in the check of falling of the interval junctions within the limits of the step and, in case of their falling, in the termination of this step at this junction. The application of the above-indicated correction will allow us to eliminate completely the influence of discontinuities of the second and higher order derivatives, since in each step we will solve the Cauchy problem the input data for which are only the coordinates and their first derivatives.

In calculations with extended precision (for example, with quadruple precision), already the coordinates themselves and their first derivatives are discontinuous; as demonstrated above, they are discontinuous approximately in the 13^{th} –17^{th} decimal place depjunctioning on the employed ephemerides. To eliminate these discontinuities, the method of ephemeris smoothing [3, 4] that allows us to correct slightly the interpolated coordinates and their derivatives so that at the junctions of the interpolation intervals they remain continuous in calculations with quadruple precision is used in the present work. This method has already been used in [3, 4] to smooth the ephemerides DE423 and DE430. However, it was little inaccurately described; therefore, below we present its correct description.

Let us first describe the method of smoothing of the American ephemerides, that is, when the coefficients for interpolation of the object coordinates are given in the ephemerides. We designate by $b_0, \ldots, b_n$ the sought-after coefficients for the corrected (smoothed) coordinate. Then the smoothed coordinate and its kth-order derivative are calculated from the formula analogous to formula (1):

$$\tilde{x}^{(k)}(\tau) = b_0 p_0^{(k)}(\tau) + \cdots + b_n p_n^{(k)}(\tau) . \tag{7}$$

The application of the examined method for any arbitrary interpolation interval consists in solution of the problem of conditional minimization

$$\Phi = \sum_{i=0}^{m} (b_i - a_i)^2 + \sum_{i=0}^{k} \left\{ \lambda_{2i+1} \left[\tilde{x}^{(i)}(-1) - x_L^{(i)} \right] + \lambda_{2i+2} \left[\tilde{x}^{(i)}(1) - x_R^{(i)} \right] \right\} \to \min , \tag{8}$$

where Φ is the Lagrange function, $x_L^{(i)}$ and $x_R^{(i)}$ are values of the ith order derivatives of the coordinate (i = 0 corresponds to the coordinate itself) assigned at the left and right junctions of the interpolation interval, $\lambda_1, ..., \lambda_{2k+2}$ are the Lagrange multipliers, m is the maximum serial number of the corrected coefficient ($m \le n$), k is the maximum order of the smoothed derivative, and $2k+1 \le m$, since the number of conditions should be less than or equal to the number of unknowns b_i (in the present work, we consider $m = 2k+1$). Here $x_L^{(i)}$ is the half-sum of the ith left and right derivatives calculated from formula (1) at the left junction of the interpolation interval, and $x_R^{(i)}$ is the analogous half-sum calculated at the right junction of the interval.

Condition of minimum (4) is the equality to zero of the partial derivatives of Φ with respect to the unknown parameters $b_0, \ldots, b_m$ and $\lambda_1, ..., \lambda_{2k+2}$:

$$\begin{cases} \dfrac{\partial\Phi}{\partial b_j} = 2(b_j - a_j) + \sum\limits_{i=0}^{k}\left[\lambda_{2i+1} p_j^{(i)}(-1) + \lambda_{2i+2} p_j^{(i)}(1)\right] = 0 \quad (j = 0, ..., m), \\ \dfrac{\partial\Phi}{\partial\lambda_{2j+1}} = \tilde{x}^{(j)}(-1) - x_L^{(j)} = 0, \qquad \dfrac{\partial\Phi}{\partial\lambda_{2j+2}} = \tilde{x}^{(j)}(1) - x_R^{(j)} = 0 \quad (j = 0, ..., k). \end{cases} \tag{9}$$

Re-designating these parameters in the same sequence by $X_0, ..., X_{m+2k+2}$, we can write system (9) in the form

$$\begin{cases} A_{00}\, X_0 + \ldots + A_{0,m+2k+2}\, X_{m+2k+2} = B_0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ A_{m+2k+2,0}\, X_0 + \ldots + A_{m+2k+2,m+2k+2}\, X_{m+2k+2} = B_{m+2k+2}. \end{cases} \tag{10}$$

This system is linear for the unknown parameters, and its nonzero coefficients are determined by the formulas

$$A_{jj} = 2 \quad (j = 0, ..., m),$$

$$A_{j,m+2l+1} = p_j^{(l)}(-1), \quad A_{j,m+2l+2} = p_j^{(l)}(1) \quad (j = 0, ..., m,\ l = 0, ..., k),$$

$$A_{m+2j+1,l} = p_l^{(j)}(-1), \quad A_{m+2j+2,l} = p_l^{(j)}(1) \quad (j = 0, ..., k,\ l = 0, ..., m),$$

$$B_j = 2a_j \quad (j = 0, ..., m),$$

$$B_{m+2j+1} = f_L^{(j)} - \sum_{i=m+1}^{n} a_i p_i^{(j)}(-1), \quad B_{m+2j+2} = f_R^{(j)} - \sum_{i=m+1}^{n} a_i p_i^{(j)}(1) \quad (j = 0, ..., k).$$

Solving system (10), we find $m+1$ coefficients $b_i (i = 0, ..., m)$. The remaining coefficients $b_{m+1}, ..., b_n$ are equal to the corresponding initial coefficients $a_{m+1}, ..., a_n$ of the ephemerides; however, the number of bits in them increases to 128 (quadruple precision).

Using the above-described method, we calculated four variants of smoothing of the ephemerides DE431 for k = 1, 2, 3, and 4, that is, up to the first, second, third, and fourth derivatives. In the last two variants, the number of the coefficients n in the smoothed ephemerides was increased to 7 for objects from Saturn to Pluto in the variant with k = 3 and to 9 in the variant with k = 4 for objects from Jupiter to Pluto; moreover, the coefficients of the initial ephemerides missing for this purpose were set equal to zero. All calculations were performed with quadruple precision.

TABLE 1. Differences between the Smoothed and Initial Ephemerides

Ephemerides	$k = 1$		$k = 2$		$k = 3$		$k = 4$	
	$\overline{\Delta r / r}$	$\overline{\Delta v / v}$	$\overline{\Delta r / r}$	$\overline{\Delta v / v}$	$\overline{\Delta r / r}$	$\overline{\Delta v / v}$	$\overline{\Delta r / r}$	$\overline{\Delta v / v}$
	$\underline{\Delta r / r}$	$\underline{\Delta v / v}$	$\underline{\Delta r / r}$	$\underline{\Delta v / v}$	$\underline{\Delta r / r}$	$\underline{\Delta v / v}$	$\underline{\Delta r / r}$	$\underline{\Delta v / v}$
DE431	$4.3 \cdot 10^{-17}$	$4.5 \cdot 10^{-15}$	$2.7 \cdot 10^{-12}$	$4.2 \cdot 10^{-11}$	$4.6 \cdot 10^{-12}$	$2.2 \cdot 10^{-10}$	$1.6 \cdot 10^{-11}$	$7.9 \cdot 10^{-10}$
	$2.1 \cdot 10^{-16}$	$1.8 \cdot 10^{-13}$	$8.7 \cdot 10^{-10}$	$2.5 \cdot 10^{-9}$	$6.0 \cdot 10^{-9}$	$1.7 \cdot 10^{-8}$	$2.1 \cdot 10^{-8}$	$6.3 \cdot 10^{-8}$
EPM2011	$2.6 \cdot 10^{-13}$	$4.3 \cdot 10^{-12}$	$1.2 \cdot 10^{-11}$	$5.6 \cdot 10^{-11}$	$7.8 \cdot 10^{-11}$	$5.5 \cdot 10^{-10}$	$5.2 \cdot 10^{-10}$	$3.3 \cdot 10^{-9}$
	$6.3 \cdot 10^{-12}$	$9.9 \cdot 10^{-11}$	$4.0 \cdot 10^{-10}$	$1.2 \cdot 10^{-9}$	$2.7 \cdot 10^{-9}$	$1.2 \cdot 10^{-8}$	$2.3 \cdot 10^{-8}$	$8.4 \cdot 10^{-8}$

The ephemerides EPM2011 were smoothed using an analogous algorithm with re-designation of some coefficients, since in these ephemerides, the coefficients for interpolation of the object velocities rather than coordinates are given. Thus, the integration constant C was used as a zero-order coefficient of the sequence a_i, and all other coefficients were normalized by multiplication by $T/2$. Hence, after smoothing (solving system (10)), the coefficient b_0 represented the corrected integration constant, and all other renormalized coefficients b_i were divided by $T/2$. As base polynomials for smoothing, instead of the Chebyshev polynomials p_i their primitive functions w_i were used. By analogy with the ephemerides DE431, four variants of smoothing were performed for the ephemerides EPM2011.

To check the difference between the smoothed and initial ephemerides, we calculated relative moduli of the differences $\Delta r / r$ and $\Delta v / v$ for the positions and velocities calculated for the initial and smoothed ephemerides. These quantities were calculated with the step $\Delta\tau = 1/128$ for all interpolation intervals. Table 1 gives their average, ($\overline{\Delta r / r}$, $\overline{\Delta v / v}$), and maximum values ($\underline{\Delta r / r}$, $\underline{\Delta v / v}$), for all intervals and objects (from Mercury to the Sun). The k value for the ephemerides EPM2011 in Table 1 has the same meaning, as for the ephemerides DE431 (the serial order of the derivatives with respect to the coordinates rather than velocities).

As can be seen from Table 1, in the case of smoothing only to the first-order derivatives ($k = 1$), the coordinates calculated for the smoothed and initial ephemerides DE431 differ only in the 16$^{\text{th}}$ decimal place. The velocities differ approximately in the 13$^{\text{th}}$–15$^{\text{th}}$ decimal places. In case of smoothing to higher order ($k \geq 2$) derivatives, the difference between the smoothed and initial ephemerides increases. Thus, for $k = 4$ the difference can already be in the 8$^{\text{th}}$ decimal place. Therefore, it is not recommjunctioned to use the ephemerides smoothed up to the second and higher order derivatives for practical purposes. Below they are used only to estimate the contribution of discontinuities of the indicated derivatives to the decrease in the precision of numerical integration of the equations of motion. The smoothed variants of the ephemerides EPM2011 differ from their initial variants stronger than for the smoothed ephemerides DE431.

To demonstrate a possible increase in the precision of numerical integration of the equations of motion, forward and backward prediction of motion was carried out for two asteroids with strongly different eccentricities of their orbits. The asteroids were chosen from the data presented on the site of the Minor Planet Center (http://minorplanetcenter.net) from objects discovered recently. They have the following orbit elements: asteroid 2014BT32 has a = 1.12 a. u., e = 0.14, and i = 8.5° (the revolution period is about one year), and asteroid 2014BH25 has a = 2.66 a. u., e = 0.69, and i = 9.6° (the revolution period is about 4 years).

To predict their motion, perturbations from nine planets and the Moon were taken into account using ephemerides DE431 and EPM2011 in their initial and smoothed variants. Calculations were performed with quadruple and conventional double precisions. The Everhart method described in [15] and refined in [16] was used for numerical integration. In calculations with double precision, the method of the 15$^{\text{th}}$ order was used, and in calculations with quadruple precision, the method of the 31$^{\text{st}}$ order was used. Prediction was performed with a variable step both with and without adjustment of the integration step to the junctions of the ephemeris intervals.

In the process of adjustment, the minimum interpolation interval, which in the ephemerides DE431 was 4 days (for the Moon), was considered, and the interpolation intervals for other objects were multiple to this value. Therefore, for ephemerides DE431, falling of the time moment with the Julian date multiple to four (after subtraction of the fractional half of the day) within the integration step was checked, and if such moment falls within the integration step,

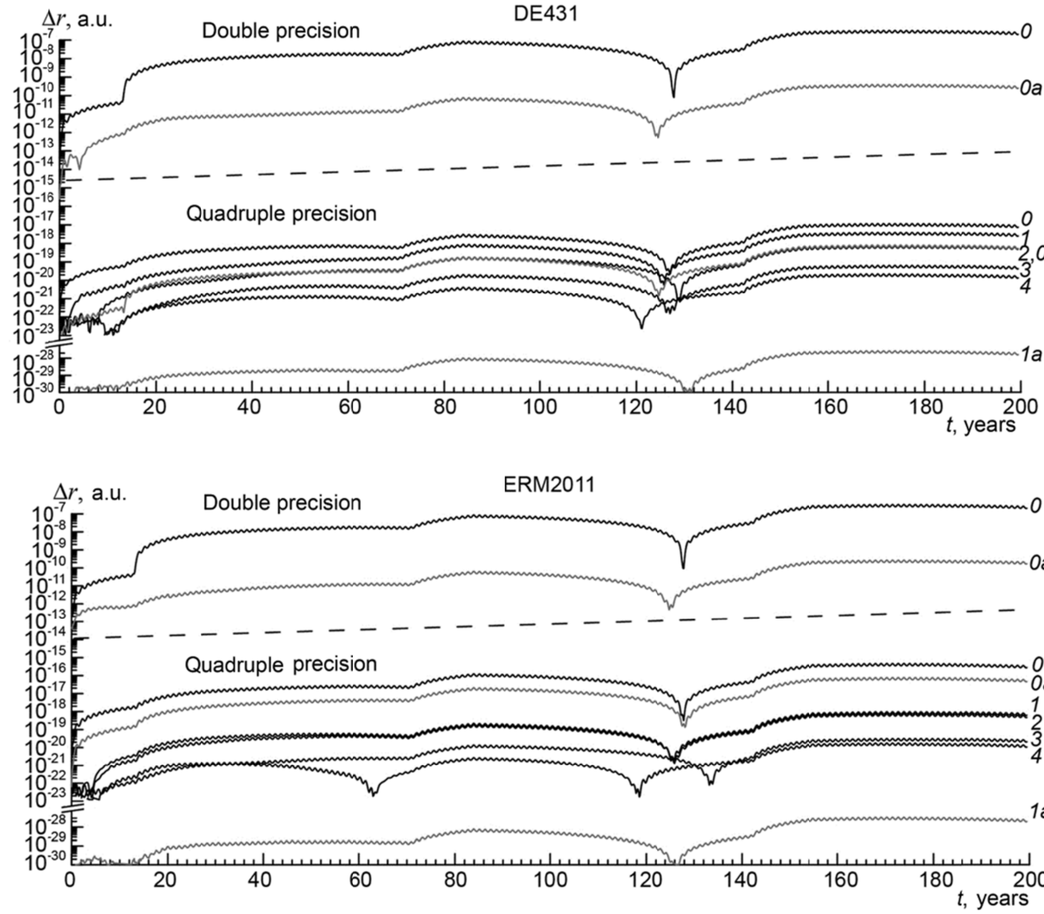

Fig. 2. Comparison of the results of forward and backward prediction of motion of asteroid 2014BT32 using the ephemerides DE431 and EPM2011.

the step ended at this very moment. In the ephemerides EPM2011, there are two incommensurable minimum interpolation intervals: 4 days (for the Moon and the Sun) and 5 days (for Mercury), and the interpolation intervals for other objects are multiple to this two minima. Therefore, in this case in the process of correction we checked falling of Julian dates multiple to four and five (after subtraction of the fractional half of the day) within the integration step.

The motion was predicted for the time interval from the initial epoch for the orbit elements (on February 5, 2014) to the junction of the period covered by the ephemerides EPM2011 (October 2214). The coordinates of object 2014BT32 were registered with a 50-day period, and the coordinates of object 2014BH25 were registered with a 200-day period; the results were delivered from the integrator output. The final date of integration was slightly changed in order that the prediction interval was multiple to these periods. Then the coordinates obtained by forward and backward integration for the same moments of time were compared by means of calculation of the modulus of their difference Δr.

Figure 2 shows the results obtained for asteroid 2014BT32, and Fig. 3 shows the results obtained for asteroid 2014BH25 for variants of prediction using:

– original ephemerides (curves *0*),

– ephemerides smoothed up to the first derivatives (curves *1*),

– ephemerides smoothed up to the second derivatives (curves *2*),

– ephemerides smoothed up to the third derivatives (curves *3*), and

– ephemerides, smoothed up to the fourth derivatives (curves *4*).

In addition, variants *0* and *1* were calculated both with and without adjustment of the integration step. In case of adjustment, letter "*a*" was added to the designation of curves, and the curves themselves and their designations were

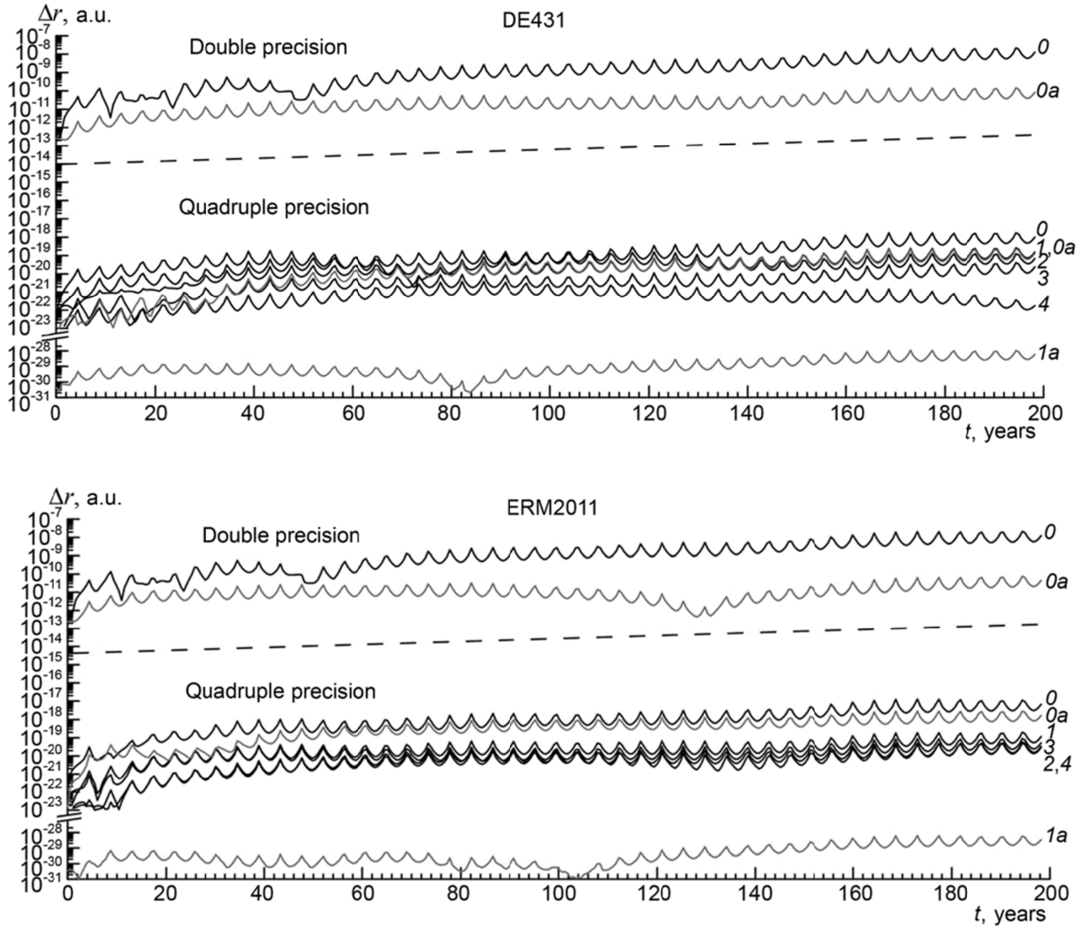

Fig. 3. Comparison of the results of forward and backward prediction of motion of asteroid 2014BH25 using ephemerides DE431 and EPM2011.

drawn in grey color. The results obtained with double and quadruple precision are separated by dashed straight lines in Figs. 2 and 3.

From Figs. 2 and 3 it can be seen that in calculations with the conventional double precision, adjustment of the integration step to the junctions of the ephemeris interpolation intervals leads to an increase in the integration precision approximately by 2–3 orders of magnitude (curves *0* and *0a*). The difference between the positions of curves *0* and *0a* calculated with quadruple precision is smaller; in this case, the precision increased on average by 1 order of magnitude. The use of the smoothed ephemerides (curves *1–4*) leads to an increase in precision by several orders of magnitude, and this increase, as a whole, is directly proportional to the maximum serial order of the smoothed derivatives, though small deviations from this depjunctionence are observed (curves *2* and *3* in Fig. 3). And finally, the maximum increase in the precision (by 10 and even more orders of magnitude) takes place in the case of joint use of the smoothed ephemerides and adjustment of the integration step (curves *1a*).

Our investigations suggest that to predict motion of asteroids with allowance for perturbations from any ephemerides of major planets and the Moon, the step of numerical integration must be adjusted to the junctions of the ephemeris interpolation intervals. This is especially true for calculations with conventional double precision; as demonstrated above, precision of integration with corresponding adjustment of the integration step can be increased by several orders of magnitude. In calculations with the quadruple precision more and more often used in heavenly mechanics, in addition to the step adjustment, it is necessary to use ephemerides in which the object coordinates and their first derivatives remain continuous at the junctions of the adjacent interpolation intervals with the preset precision.

Such ephemerides together with step adjustment allow the motion of asteroids to be predicted with the highest possible precision provided by 128-bit floating-point numbers. It is obvious that such ephemerides in themselves must have 128 bits, that is, comprise the coefficients calculated with quadruple precision. Therefore, it would be desirable to address the developers of modern ephemerides to publish ephemerides calculated with quadruple precision and to retain with the same precision the continuity of object coordinates and first derivatives. The continuity of higher-order derivatives is not necessary, since the influence of their discontinuities on the precision of numerical integration is completely eliminated by adjustment of the integration step size to the junctions of the ephemeris interpolation intervals.

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