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# Enhancing the Accuracy of Numerical Integration of the Equations of Asteroid Motion with Perturbations from Major Planets and the Moon from the DE Ephemerides

**A. P. Baturin***

*Scientific Research Institute of Applied Mathematics and Mechanics, Tomsk State University, Tomsk, 634050 Russia*
**e-mail: alexbaturin@sibmail.com*

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**Abstract**—The discontinuous behavior of coordinates of planets and the Moon and their derivatives, which are determined from their modern ephemerides, at the boundaries of adjacent interpolation intervals is illustrated using the example of the DE436 ephemerides. The numerical integration of the equations of motion of two asteroids demonstrates that the integration accuracy increases by several orders of magnitude if the step of numerical integration is matched to the boundaries of ephemeris interpolation intervals. In addition, an algorithm for ephemeris smoothing at the boundaries of interpolation intervals is developed and applied in order to eliminate the jumps of coordinates and their first-order derivatives emerging in extended- and quad-precision calculations. This algorithm allows one to remove the jumps of coordinates and their derivatives up to any given order. It is demonstrated that the use of ephemerides smoothed to the first-order derivatives in quad-precision calculations increases the accuracy of numerical integration by ~10 orders of magnitude.

*Keywords:* asteroids, numerical integration, DE436, jumps of coordinates and derivatives, quadruple precision
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This report presents the results of studies (Baturin, 2012, 2014; Baturin and Votchel', 2014) focused on enhancing the accuracy of numerical integration of the equations of asteroid motion with perturbations from major planets and the Moon derived from the JPL (Jet Propulsion Laboratory, United States) ephemerides (ftp://ssd.jpl.nasa.gov/pub/eph/planets).

It is known that the continuity of coordinates and their first-order derivatives at the boundaries of adjacent interpolation intervals is preserved in the modern ephemerides of major planets and the Moon, while second-order and higher-order derivatives are discontinuous. In addition, the coefficients in Chebyshev polynomials are given in all the currently available ephemerides with just 16 decimal digits (i.e., as double-precision numbers). However, extended-precision calculations are now used more and more often (Chernitsov and Baturin, 2001; Syusina et al., 2012; Baturin, 2016; Chernitsov et al., 2016). The interpolated coordinates and their first-order derivatives are no longer continuous in such calculations: they undergo jumps in the 15th or the 16th decimal digit at the boundaries of ephemeris intervals.

Such jumps are naturally manifested at the right-hand sides of differential equations characterizing the asteroid motion, since these right-hand sides depend continuously on the coordinates of perturbing bodies. Therefore, they are also discontinuous at the boundaries of ephemeris intervals. More specifically, the second-order and higher-order derivatives are discontinuous in double-precision calculations, and the right-hand sides themselves and the first-order and higher-order derivatives undergo jumps in extended-precision calculations. In both cases, this leads to a reduction in the accuracy of numerical integration of the equations of motion, since the continuity of functions on the right-hand sides and their smoothness up to an arbitrary order is generally presumed in the numerical integration techniques.

Two methods (Baturin, 2012, 2014; Baturin and Votchel', 2014) for suppressing the influence of the mentioned jumps on the accuracy of numerical integration of the equations of motion are used in the present study. The first approach (ephemeris "smoothing") consists in correcting the coefficients in Chebyshev polynomials in such a way as to preserve the continuity of coordinates, which are derived from ephemerides, and their derivatives up to a certain given order at the boundaries of adjacent integration intervals in extended- and quad-precision calculations. The second method involves matching the integration step to the boundaries of interpolation intervals and eliminates the influence of discontinuities of the second-order and higher-order coordinate derivatives on the accuracy of numerical integration.

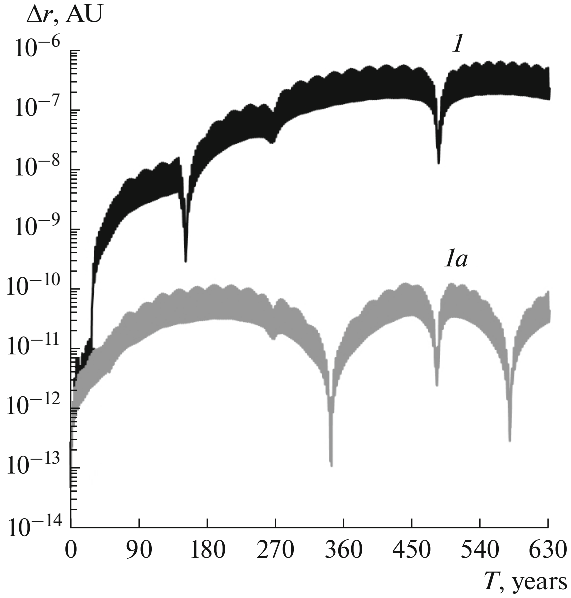


**Fig. 1.** Comparison of the results of direct and reverse integration with a 64-bit grid for 2017 FN128.

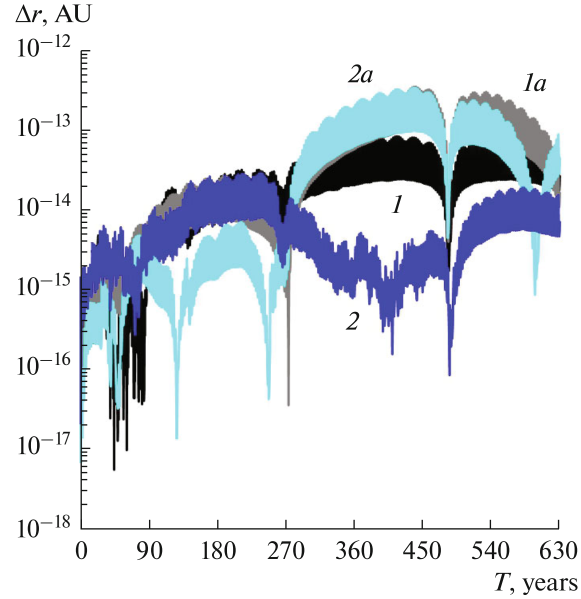


**Fig. 2.** Comparison of the results of direct and reverse integration with an 80-bit grid for 2017 FN128.

The direct and reverse integration of the motion of two asteroids (2017 FN128 with $a = 1.84$ AU, $e = 0.55$, and $i = 61.1°$ and 2017 FO63 with $a = 2.37$ AU, $e = 0.84$, and $i = 2.1°$) having several close encounters with planets was performed in order to demonstrate the enhancement of the accuracy of numerical integration of the equations of motion. It should be stressed that these are just model examples illustrating the advantages of the above approaches; actual prediction of asteroid motion is beyond the scope of the present study.

Perturbations from nine planets and the Moon derived from the initial and smoothed versions of the DE436 ephemerides were taken into account in the integration. Calculations were carried out with 64-bit (double precision, 16 decimal digits), 80-bit (extended precision, 19 decimal digits), and 128-bit (quadruple precision, 34 decimal digits) grids. The Everhart algorithm (Everhart, 1985) was applied, and its order was set to 15, 19, or 31 depending on the bit grid used. The numerical integration was performed for a time interval from the initial epoch of orbital elements (March–April 2017) to the end of the period covered by the DE436 ephemerides (January 2650). The results were output in 100-day intervals, and the coordinates obtained in the direct and reverse integration at the same moments in time were then compared by calculating the magnitude of their difference ($\Delta r$).

Figure 1 presents the plots of $\Delta r$ obtained in the process of direct and reverse integration of the motion of 2017 FN128 with a 64-bit grid.

Curve *1* in Fig. 1 (and other figures) corresponds to the initial DE436 ephemerides, while curve *1a* represents the results obtained by correcting the integration step to match it to the boundaries of ephemeris interpolation intervals. It can be seen that this correction results in a considerable (3–4 orders of magnitude) enhancement of accuracy of numerical integration of the equations of motion.

Figure 2 presents the results of similar calculations with an 80-bit grid. Curve *2* in Fig. 2 (and other figures) corresponds to the smoothed DE436 ephemerides.

It can be seen that the accuracy of integration with the 80-bit grid is significantly higher than that with the 64-bit grid (Fig. 1), but the step correction and ephemeris smoothing provide almost no accuracy enhancement.

Figure 3 shows the results of calculations with a 128-bit grid.

It can be seen that the 128-bit grid offers a considerable advantage over the 64- and 80-bit grids in terms of accuracy of numerical integration (Figs. 1 and 2). However, the step correction does not result in any significant changes in this accuracy (curves *1* and *1a* are almost coincident). The smoothing of ephemerides (curve *2*) also has no effect on the accuracy, and only the use of smoothed ephemerides with step correction (curve *2a*) increases the accuracy of numerical integration by ~10 orders of magnitude.

The results of similar calculations for 2017 FO63 are presented in Figs. 4–6.

These results agree qualitatively with those obtained for 2017 FN128 (Figs. 1–3), although certain quantitative differences may be noted. For example, the overall reduction in the accuracy of numerical integration for 2017 FO63 is somewhat larger than that

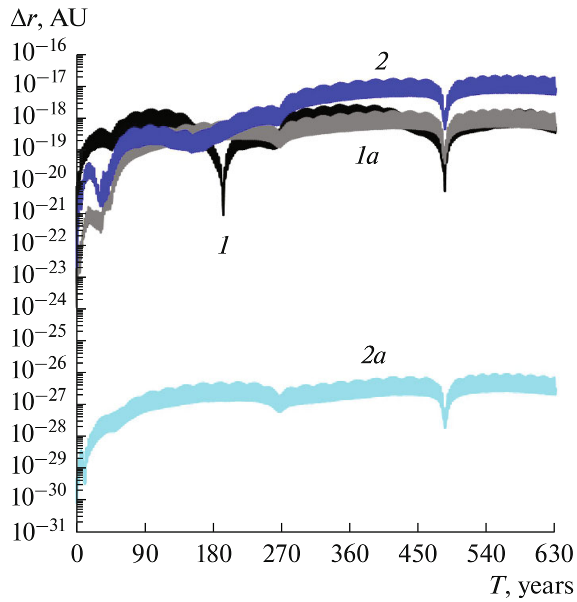


**Fig. 3.** Comparison of the results of direct and reverse integration with a 128-bit grid for 2017 FN128.

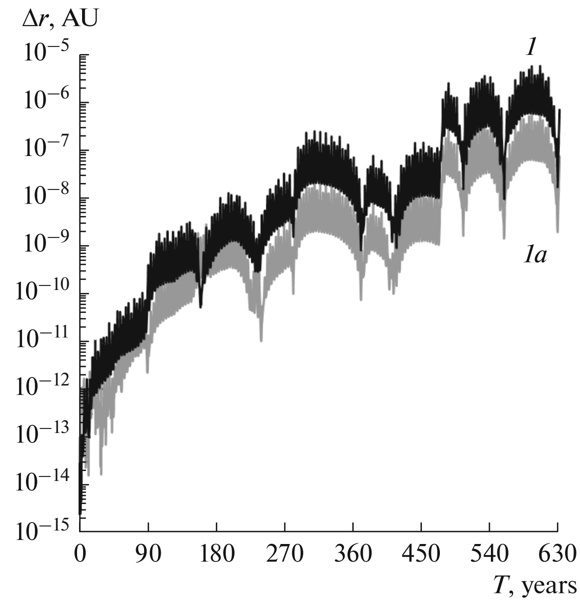


**Fig. 4.** Comparison of the results of direct and reverse integration with a 64-bit grid for 2017 FO63.

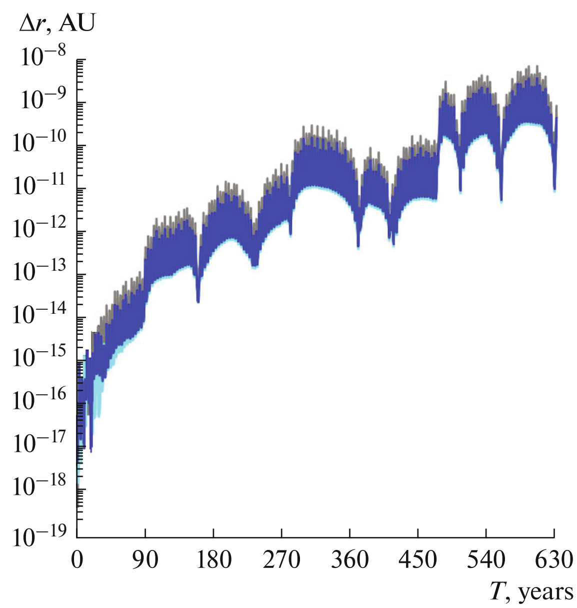


**Fig. 5.** Comparison of the results of direct and reverse integration with an 80-bit grid for 2017 FO63.

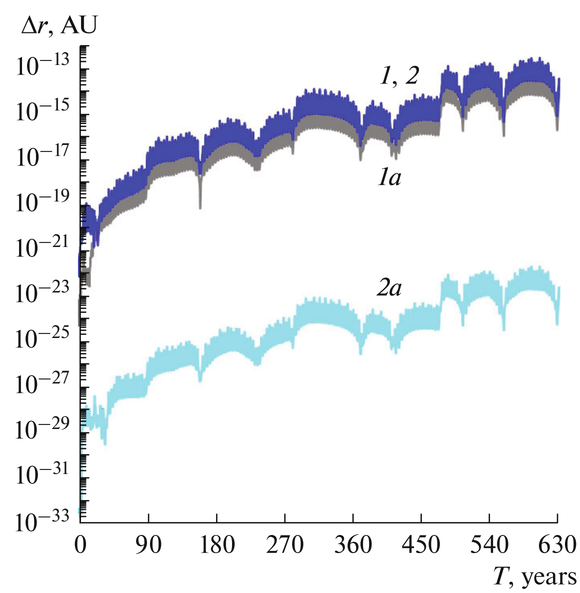


**Fig. 6.** Comparison of the results of direct and reverse integration with a 128-bit grid for 2017 FO63.

for 2017 FN128, which may be attributed to the fact that the orbit eccentricity of the former asteroid is considerably higher. The introduction of the step correction in integration with the 64-bit grid (Fig. 4) results in just an order-of-magnitude accuracy enhancement. If the 80-bit grid is used (Fig. 5), the results obtained using the initial and smoothed ephemerides and with and without step correction are essentially the same but are three orders of magnitude more accurate than the ones obtained using the 64-bit grid (Fig. 4). In quad-precision calculations (Fig. 6), the smoothed ephemerides and the step correction applied individually do not provide any significant accuracy enhancement (the same is true for 2017 FN128; see Fig. 3).

However, when they are introduced simultaneously (curve *2a*), the accuracy increases by ~10 orders of magnitude.

The results suggest that the integration step should be matched to the boundaries of ephemeris interpolation intervals if the equations of asteroid motion are integrated numerically with perturbations from major planets and the Moon derived from certain ephemerides. This is especially important for common double-precision calculations: it was demonstrated above (Figs. 1 and 4) that the accuracy of integration with a properly corrected step may increase by several orders of magnitude.

In extended-precision calculations (80-bit grid), the step correction and smoothed ephemerides provide almost no enhancement of the accuracy of numerical integration (Figs. 2 and 5). Apparently, the enhancement actually occurs, but its magnitude is roughly the same (~3 orders of magnitude) as the one associated with the transition from a 64-bit grid to the 80-bit one. As a result, the accuracy enhancement attributable to the step correction and the smoothing of ephemerides is "screened" by the accuracy enhancement due to the grid change and becomes indiscernible.

In quad-precision calculations, which are used more and more often in celestial mechanics, one should apply the step correction and use ephemerides with the continuity of coordinates and their first-order derivatives preserved at the boundaries of adjacent interpolation intervals. This provides an opportunity to integrate the equations of motion with the maximum possible accuracy provided by a 128-bit grid. It can be seen from Figs. 3 and 6 that this accuracy is approximately 10 orders of magnitude higher than the accuracy of integration with the initial ephemerides. Naturally, the ephemerides themselves should also be 128-bit (i.e., contain quad-precision coefficients).

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